8	9	10	90	91
9	10	11	110	111
10	11	12	132	133
11	12	13	156	157
12	13	14	182	183
13	14	15	210	211
14	15	16	240	241
15	16	17	272	273
16	17	18	306	307
17	18	19	342	343
18	19	20	380	381
19	20	21	420	421
20	21	22	462	463
etc.	etc.	etc.	etc.	etc.

94. Proposed v L. C. WALKER, A. M., Petaluma High School. Petaluma, Cal.

Show that the area of a rational triangle cannot be a square number.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let a^2+b^2, a^2+c^2, b^2+c^2 be the sides.

$\therefore$ Area$=abc\sqrt{(a^2+b^2+c^2)}$.

Let $a=m^2+mn$, $b=mn+n^2$, $c=mn$.

$\therefore$ Area$=mn(m^2+mn)(n^2+mn)(m^2+n^2+mn)$.

Let $n=1$. $\therefore$ Area$=m^2(m+1)^2(m^2+m+1)$.

For integral values of m, m^2+m+1 is not a square.

Let $9p$, $10p$, $17p$ be the sides. Then area$=36p^2$.

Let $3p$, $25p$, $26p$ be the sides. Then area$=36p^2$.

This gives two series of triangles whose areas are square numbers, thus proving that the proposition is not true.

95. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

There are two unequal square numbers the sum of whose sum, difference, product, and quotient, is a square. Find the two numbers.

This is problem 91, so numbered by mistake. Its solution is in the April number, page 113. ED.

AVERAGE AND PROBABILITY.

115. Proposed by L. C. WALKER, A. M., Petaluma High School, Petaluma, Cal.

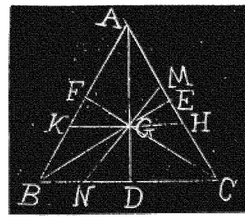
Three points are at random within a given triangle. Find the chance that they will all lie on one side of some one line that can be drawu through the center of gravity of the triangle.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa., and J. R. HITT, Coronal Institute, San Marcos, Tex.

Project the given triangle into an equilateral triangle, side $=a$; let G be the center of gravity, CH or $CM=x$.

Then $BK=\dfrac{a(a-2x)}{2a-3x}$, $AK=\dfrac{a(a-x)}{2a-3x}$, $AH=a-x$.

Area $AKH=\dfrac{\triangle(a-x)^2}{a(2a-3x)}$, $BN=\dfrac{2ax-a^2}{3x-a}$,



$$CN=\frac{ax}{3x-a}.$$

Area $CMN=\dfrac{\triangle x^2}{a(3x-a)}$, area $ABNM=\triangle\left[1-\dfrac{x^2}{a(3x-a)}\right]$.

$$p=\left[\int_0^{\frac{1}{2}a}\frac{(AKH)^3dx}{\triangle^3}+\int_{\frac{1}{2}a}^a\frac{(ABNM)^3dx}{\triangle^3}\right]\Big/\int_0^{\frac{1}{2}a}dx$$

$$=\frac{2}{a}\int_{\frac{1}{2}a}^a\left[1-\frac{x^2}{a(3x-a)}\right]^3dx+\frac{2}{a^4}\int_0^{\frac{1}{2}a}\frac{(a-x)^6dx}{(2a-3x)^3}$$

$$=\frac{2}{a}\int_{\frac{1}{2}a}^a\left[1-\frac{x^2}{a(3x-a)}\right]^3dx+\frac{2}{a^4}\int_{\frac{1}{2}a}^a\frac{x^6dx}{(3x-a)^3}$$

$$=\frac{2}{a}\int_{\frac{1}{2}a}^a dx-\frac{6}{a^2}\int_{\frac{1}{2}a}^a\frac{x^2dx}{3x-a}+\frac{6}{a^3}\int_{\frac{1}{2}a}^a\frac{x^4dx}{(3x-a)^2}$$

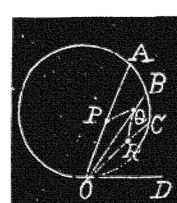
$$=\tfrac{1}{27}\left(\tfrac{23}{2}-\tfrac{20}{3}\log 2\right).$$

116. Proposed by the late ENOCH BEERY SEITZ.

The average area of the quadrilateral formed by joining four random points on the surface of a circle, radius a, is $4a^2/3\pi$.

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics. The Temple College, Philadelphia, Pa.

Let a=radius of given circle, A=its area, $\triangle$=required average, $\triangle'$=the average area when the four points are taken on both the circle A and a concentric annulus B, $\triangle_1$=the average area when three points are taken on A and one on B.



Then $(\triangle'-\triangle)A=4B(\triangle_1-\triangle)$.

But $\triangle:\triangle'=A:A+B$.

$\therefore\ \triangle'=\dfrac{(A+B)\triangle}{A}$. $\quad\therefore\ \triangle=\frac{4}{5}\triangle_1$.

Let one point O be on the circumference of the circle, and the other points P, Q, R anywhere on its surface. Let $OP=x$, $OR=y$, OQ